Bohr’s power series theorem and local Banach space theory

Abstract. How large can the sum of the moduli of the terms of a convergent power series be? In 1914 Harald Bohr published the following surprising result: Suppose that $|\sum_{k} a_k z^k| < 1$ for each $z$ in the unit disk. Then $\sum_{k} |a_k z^k| < 1$ when $|z| < \frac{1}{3}$, and moreover the radius $\frac{1}{3}$ is best possible. Recently, several authors studied Bohr’s power series theorem in higher dimensions: Given a domain $B$ in $\mathbb{C}^n$, what is the largest radius $K(B)$ (the so-called Bohr radius of $B$) such that if $|\sum_{\alpha} c_{\alpha} z^{\alpha}| < 1$ for all $z \in B$, then $\sum_{\alpha} |c_{\alpha} z^{\alpha}| < 1$ whenever $z \in K(B) B$? A result of Dineen-Timoney and Boas-Khavinson states that for the $n$-dimensional polydisc $B_{\ell^2_n}$ the scalar sequence $(K(B_{\ell^2_n}))$ tends to zero in the dimension $n$, and that its decay is essentially like $\frac{1}{\sqrt{n}}$. We link this cycle of ideas around multi-variable power series with local Banach space theory, in particular with our recent research on unconditionality in spaces of $m$-homogeneous polynomials on Banach spaces.