Hilbert transform along $C^{1+\epsilon}$ vector filed

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Abstract

Let $v$ be a vector field from $\mathbb{R}^2$ to the unit circle $S^1$. We study the operator

$$H_v f(x) = \mathrm{p.v.} \int_{-1}^{1} f(x - tv(x)) \frac{dt}{t}.$$  

We prove that if the vector field $v$ has $1 + \epsilon$ derivatives, then $H_v$ extends to a bounded map from $L^2$ into itself.