The well-known Banach-Tarski paradox states that the unit ball in $\mathbb{R}^3$ can be partitioned into finitely many pieces that can be rearranged by rotations and translations to form two unit balls. More than simply a curiosity, this type of paradox is intimately tied to the important group-theoretic notion of amenability. It also has applications in measure theory, for example, as part of Drinfeld, Margulis, and Sullivan’s theorem that Lebesgue measure is the unique finitely additive rotation-invariant measure on the Lebesgue measurable subsets of the $n$-sphere, for $n > 1$. I will discuss some recent developments in descriptive graph combinatorics which have applications to the problem of how pathological the pieces in these paradoxes must be.